

# Resonance Frequency Dependence Of A1 Lamb Mode On The Pitch Of The Electrode Structure

V. Plessky, S. Yandrapalli, S. Küçük and L.G. Villanueva

ANEMS Laboratory,  
EPFL

Lausanne, Switzerland  
[victor.plessky@gmail.com](mailto:victor.plessky@gmail.com)

**Abstract**—Dispersion of A1 Lamb mode near onset frequency is discussed. This mode is close to that used in XBAR devices. The typically used approximation is based on elliptic form of slowness curve and ignoring mode interaction is not valid for widely used materials, such as ZY-LiNbO<sub>3</sub>. Alternative phenomenological dispersion formula is proposed.

**Keywords**—A1 Lamb mode, XBARs, dispersion

## I. INTRODUCTION

The resonance frequency of recently proposed XBARs [1] slightly depends on the pitch  $p$  of the electrode structure [2]. At the onset frequency, for large pitch  $p = L \gg t$  ( $t$  is the thickness of crystalline membrane) the wave is presented as a bulk shear wave with the wavelength  $\lambda_T = 2p$  bouncing up and down, the small longitudinal component being ignored. However, for finite wavelength, the anti-symmetric Lamb A1 mode has dispersion and its frequency increases. Here we present an explanation of this dependence, formulas for isotropic membrane and comparison with FEM-simulated data.

Usually the effect of finite period (Fig.1, Fig.2b) is described by introducing fixed wavelength in horizontal x-direction:  $\lambda_L = 2L$ , which results in the increased resonance frequency.

$$f_R = \sqrt{\left(\frac{V_T}{2d}\right)^2 + \left(\frac{V_L}{2L}\right)^2} \approx \frac{V_T}{2t} \cdot \left(1 + 0.5 \left(\frac{V_L}{V_T}\right)^2 \cdot \left(\frac{d}{L}\right)^2\right) \quad (1)$$

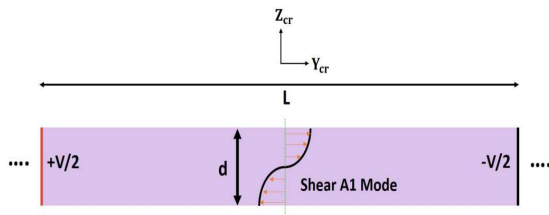


Fig. 1. Rectangular Shear Bulk Acoustic resonator model

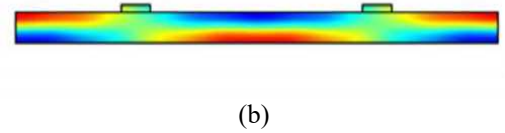
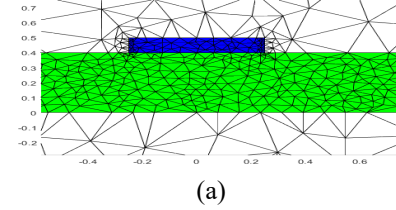


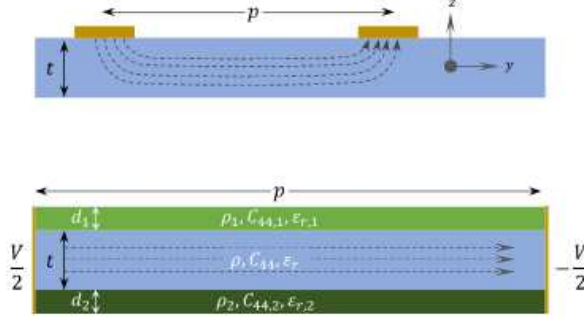
Fig. 2. a) Dense FEM grid; b) Horizontal displacements in XBAR

## II. METHODS&RESULTS

The analytic derivation of Lamb wave dispersion in anisotropic piezoelectric media seems to be impossible, therefore we consider here the isotropic platelet. By simplifying the classic dispersion equation [3] for anti-symmetric Lamb mode and using  $d/L$  as a small parameter we get the approximate dispersion equation:

$$\frac{\Delta\omega}{\omega_0} = \left(\frac{t}{L}\right)^2 \cdot \left\{ \frac{1}{2} + \frac{8}{\pi} \cdot \frac{V_T}{V_L} \cdot \frac{1}{\tan(\pi \frac{t}{\lambda_L})} \right\} \quad (2)$$

valid up to  $(t/L)^2$  order. Here, the first term “1/2” in brackets corresponds to the above described simple “shear wave in rectangular resonator” model, while the second term shows that longitudinal component of Lamb wave does matter. The  $\lambda_L$  is the wavelength of longitudinal bulk wave dependent on frequency. The value of  $\tan(\pi \frac{t}{\lambda_L})$  can be different (even negative) depending on how close is the longitudinal resonance onset frequency from our A1 resonance. That explains why a kind of parabolic dependence of frequency  $\omega$  on “horizontal” wavenumber  $k$  looks so different for different Lamb modes [3] and the group velocity can even be negative near the onset frequency of a mode.



a) – top , b) - bottom

Fig. 3. One period of XBAR structure (a), simplified model (b) with electrodes on sides and horizontal electric field

We have simulated using 2D FEM software the periodic structure shown in Fig. 3a. The LiNbO3 membrane (ZY-cut) thickness is 400nm, Al electrodes width 1000nm, their thickness 100nm. The simulation gives a kind of hyperbolic dependence (Fig.4) [2]. However, when the resonance frequency deviation from the onset frequency is plotted as a function of  $t/(2p)$ , Fig.5, which is expected to be parabolic, we see a strong linear component. Moreover, re-plotting the data from Fig.4 in the same coordinates, as a function of  $t/(2p)$  we see only linear dependence.

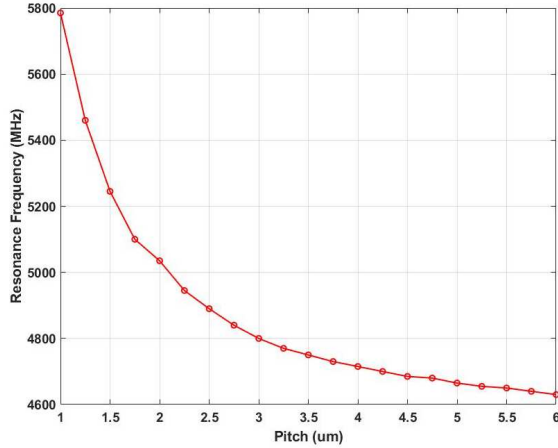


Fig. 4. XBAR structure (Fig.3a), simulated resonance frequency dependence on the pitch

### III. DISCUSSION

There can be a few reasons for non-parabolic dependence of the resonance frequency near the mode onset. Fig. 6 shows the slowness ( $1/V$ ) curves for ZY plane of the lithium niobate. In Z-direction two shear mode degenerate, have the same velocity, but the curves are not parabolic/elliptic near the top (Fig.6). Another reason can be that in simulations of the XBAR structure (Fig.3a) we changed only the pitch, not scaling the other dimensions correspondingly.

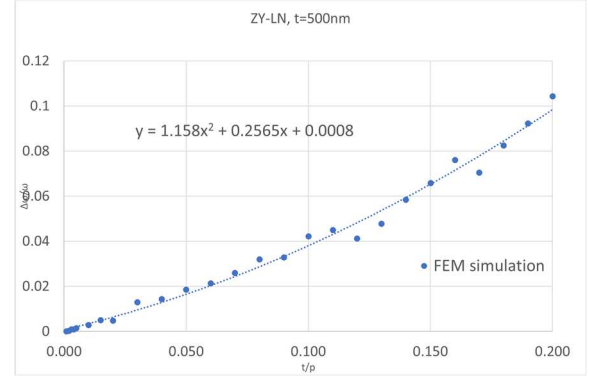
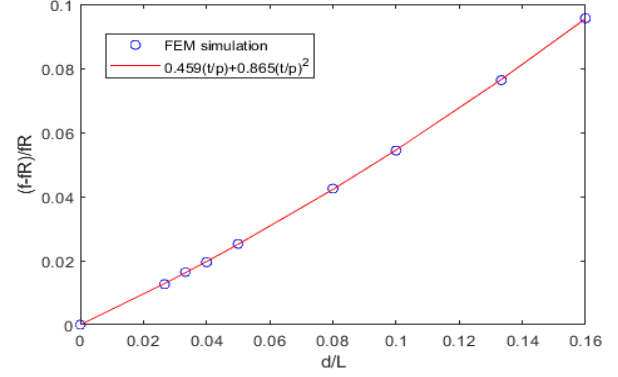


Fig. 5. Theoretical resonance frequency dependence on  $(t/2p)$ : a) realistic geometry of Fig.3a b) Simplified 1D resonator, Fig 3b.

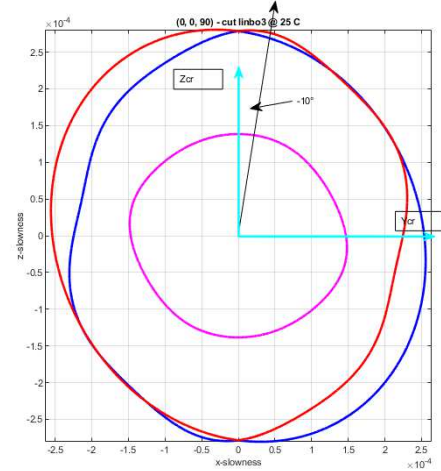


Fig. 6. Slowness curves for ZY-LN.

For practical design of XBARS we propose using the empirical formula:

$$\frac{\Delta\omega}{\omega_R} = C_1 \cdot \frac{t}{p} + C_2 \left(\frac{t}{p}\right)^2 \quad (3)$$

with linear and quadratic coefficients determined from FEM simulations and/or experiments.

#### IV. CONCLUSIONS

- The XBAR resonance frequency slightly depends on the interdigitated electrode pitch, being mainly dependent on the membrane thickness
- Often used approximation,  $f_R = \sqrt{\left(\frac{v_T}{2d}\right)^2 + \left(\frac{v_L}{2d}\right)^2}$  based on elliptic slowness curve  $k_x^2 + k_y^2 = k^2$ , is not exact
- Proposed approximation includes significant linear term:  
$$\frac{\Delta\omega}{\omega_R} = C_1 \cdot \frac{t}{p} + C_2 \left(\frac{t}{p}\right)^2$$
- Influence of close on frequency scale other Lamb modes explains different value of the parabolic coefficient and even negative group velocity in some cases

#### REFERENCES

- [1] Plessky, V., S. Yandrapalli, P. J. Turner, L. G. Villanueva, J. Koskela, and R. B. Hammond. "5 GHz laterally-excited bulk-wave resonators (XBARs) based on thin platelets of lithium niobate." *Electronics Letters* 55, no. 2 (2019): 98-100.
- [2] S. Yandrapalli, V. Plessky, J. Koskela, V. Yantchev, P. Turner and L. G. Villanueva, "Analysis of XBAR resonance and higher order spurious modes," *2019 IEEE International Ultrasonics Symposium (IUS)*, 2019, pp. 185-188, doi: 10.1109/ULTSYM.2019.8925993.
- [3] Royer, Daniel, and Eugene Dieulesaint. *Elastic waves in solids I: Free and guided propagation*. Springer Science & Business Media, 1999.